STATS 101
Introductory Statistics

Lecture 3

Elementary Probability

ELEMENTARY PROBABILITY

A **RANDOM EXPERIMENT** is an experiment such that

- The OUTCOME of the experiment is unknown.
- The COLLECTION OF ALL POSSIBLE OUTCOMES - called the **SAMPLE SPACE (S)** of the random experiment - is known.
- The experiment can be repeated indefinitely, under similar conditions.

Examples:

1. Toss a FAIR COIN:
   sample space S = {H, T}, #(S) = 2

2. Toss a fair coin 2 times:
   S = {(H,H), (H,T), (T,H), (T,T)}, #(S) = 4

3. Toss a fair coin 3 times:

S = {(H,H), (H,T), (T,H), (T,T)}x (H,T)

= {(H,H,H), (H,H,T), (H,T,H), (H,T,T),
(T,H,H), (T,H,T), (T,T,H), (T,T,T)}}

#(S) = 8

4. Roll a fair die: S = {1, 2, 3, 4, 5, 6},

#(S) = 6

5. Roll a pair of fair dice:

S = { (1,1), (1,2), …, (1,6),
(2,1), (2,2), …, (2,6),
…..
(6,1), (6,2), …, (6,6)}

#(S)=6x6=36

In all of the above examples, the sample space S is a FINITE SET, i.e., we can count its number of elements. In the following examples, S is INFINITE:

6. The BP measurements of a person

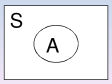
7. The TEMPERATURE of a patient

8. The amount of rainfall at a given place in a given year

SET THEORY – NOTATIONS AND TERMINOLGY

Let S = sample space of a random experiment

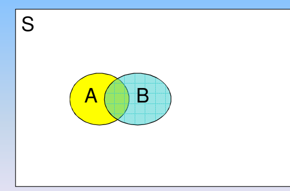
1. An EVENT A is a SUBSET of S.



2. The **UNION** of two sets A and B is

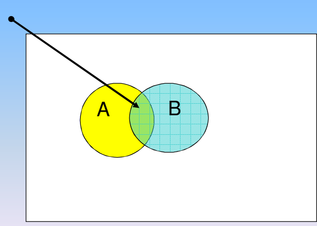
$A \cup B = \{x \in S : x \in A$ or $x \in B$ or $x \in$ both sets$\}$

$\in$: belongs to



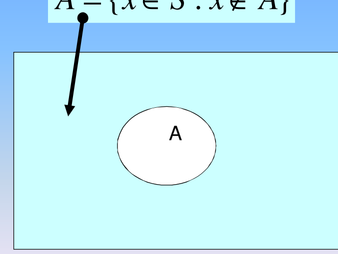
3. The **INTERSECTION** of two sets A and B is

$A \cap B = \{x \in S : x \in A$ and $x \in B\}$



4. The **COMPLEMENT** of a set A is

$\overline{A} = \{x \in S : x \notin A\}$



Example : S = {a, b, c, d, e, f, g, h}
A = {a, c, e, g}, B = {b, c, e, f}, C= {b, d}

$A \cup B$ = {a, b, c, e, f, g}

$A \cap B$ = {c, e}

$\overline{A}$ = {b, d, f, h}

$A \cap C = \phi$ A, C are DISJOINT or MUTUALLY EXCLUSIVE

Calculating Probabilities when S is FINITE

P(A) = # of points in event A/# of points in sample space S

= #(A)/#(S)

Example of Probability Calculation

Roll 2 fair dice:

P(Sum = 2) = P(1,1) = 1/36

P(Sum = 7) = P{(1,6), (2,5), (3,4),

(4,3) ,(5,2) ,(6,1)}

= 6/36 = 1/6

The following PROBABILITY DISTRIBUTION can be similarly obtained.

Probability Distribution of the Sum on Two Faces When a Pair of Fair Dice are Rolled.

SUM	# OF WAYS TO GET THE SUM	Probability
2	(1,1)	1/36
3	(1,2) (2,1)	2/36
4	(1,3) (2,2) (3,1)	3/36
5	(1,4) (2,3) (3,2) (4,1)	4/36
6	(1,5) (2,4) (3,3) (4,2) (5,1)	5/36
7	(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)	6/36
8	(2,6) (3,5) (4,4) (5,3) (6,1)	5/36
9	(3,6) (4,5) (5,4) (6,3)	4/36
10	(4,6) (5,5) (6,4)	3/36
11	(5,6) (6,5)	2/36
12	(6,6)	1/36