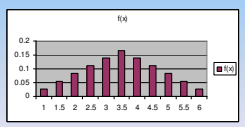
STATS 101
Introductory Statistics

Lecture 4

Discrete Probability Distributions



Random Variables

- A random variable (rv) is a function defined on elements of a sample space S.

Example 1: Suppose **3 fair coins are tossed.**
Let X = # of Heads in 3 tosses.

S = {(H, H, H), (H, H, T),
(H, T, H), (T, H, H),
(H, T, T), (T, H, T),
(T, T, H), (T, T, T) }

↓ X

{ 3, 2, 2, 1, 1, 1, 1, 0 }

In words: a random variable (rv) is a numerical characteristic X measured in a random experiment; its value is not known in advance.

Probability density function (pdf) of a DISCRETE rv

$$f(x) = P\{X = x\}$$

Example 1 (continued):
X = #(H's) in 3 tosses of a fair coin

This rv X can take values: {0, 1, 2, 3}

The probability distribution function (pdf) of X is given in the following table:

x	sample points s	f(x)
0	{(T, T, T)}	1/8
1	{(H, T, T), (T, H, T), (T, T, H)}	3/8
2	{(H, H, T), (T, H, H), (H, T, H)}	3/8
3	{(H, H, H)}	1/8

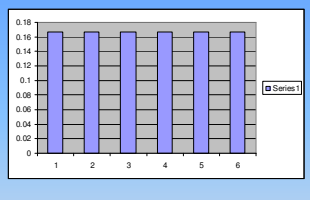
SOME COMMON DISCRETE PROBABILITY DISTRIBUTIONS

Example 1: A fair die is rolled,
let X = number on the die

Values of X are: {1, 2, 3, 4, 5, 6}

pdf of X is f(x) = P(X = x), x = 1, 2, …, 6

x	f(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6



DISCRETE UNIFORM DISTRIBUTION

POPULATION MEAN AND POPULATION VARIANCE

μ=Average of X = E(X) = $\sum xf(x)=1(1/6)+\ldots+6(1/6)=3.5$

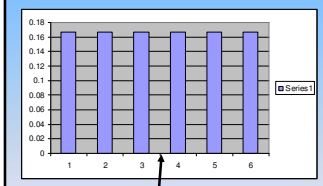
σ^2=Average of $(X-\mu)^2 = \sum(x-\mu)^2 f(x)$ =

$$\frac{[(1-3.5)^2+(2-3.5)^2+(3-3.5)^2+(4-3.5)^2+(5-3.5)^2+(6-3.5)^2}{6}$$

$=2.92$

POPULATION STANDARD DEVIATION (sd) σ =

$$\sqrt{2.92}=1.71$$



$\mu - \sigma$ $\mu = 3.5$ $\mu + \sigma$

μ= measure of CENTER POINT

σ = measure of VARIABILITY or SPREAD

- The probability distribution of X = # on a fair die is the UNIFORM distribution shown above .
- Next – probability distribution of X = sum of 2 faces when 2 fair dice are rolled

Example 2:

Suppose 2 fair dice are tossed.
Let
X = sum on the 2 faces
All possible values of X are:

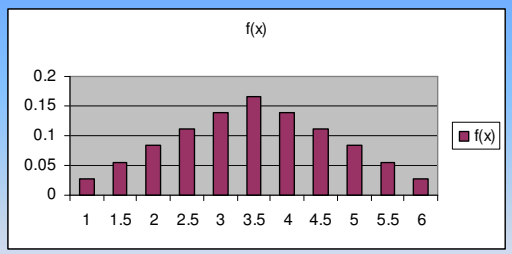
{2, 3, 4, …, 11, 12}

x		f(x)
2	{(1,1)}	1/36
3	{(1,2), (2,1)}	2/36
4	{(1,3), (2,2), (3,1)	3/36
5	{(1,4), (2,3), (3,2), (4,1)}	4/36
6	{(1,5), (2,4), (3,3), (4,2), (5,1)}	5/36
7	{(1,6), (2,5), (3,4), (4,3), (5,2), (6,	6/36
8	{(2,6), (3,5), (4,4), (5,3), (6,2)}	5/36
9	{(3,6), (4,5), (5,4), (6,3)}	4/36
10	{(4,6), (5,5), (6,4)}	3/36
11	{(5,6), (6,5)}	2/36
12	{(6,6)}	1/36

Distribution of average = (X1 + X2)/2

average = x/2		probability
1	{(1,1)}	1/36
1.5	{(1,2), (2,1)}	2/36
2	{(1,3), (2,2), (3,1)	3/36
2.5	{(1,4), (2,3), (3,2), (4,1)}	4/36
3	{(1,5), (2,4), (3,3), (4,2), (5,1)}	5/36
3.5	{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}	6/36
4	{(2,6), (3,5), (4,4), (5,3), (6,2)}	5/36
4.5	{(3,6), (4,5), (5,4), (6,3)}	4/36
5	{(4,6), (5,5), (6,4)}	3/36
5.5	{(5,6), (6,5)}	2/36
6	{(6,6)}	1/36

Distribution of average = (X1 + X2)/2



NOTE: distribution of average is 'bell shaped'

In both the examples presented in this lecture, the

random variables were DISCRETE, i.e., X was integer-valued. In the next lecture, we will be going over CONTINUOUS random variables and their probability distributions.