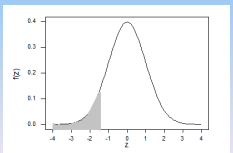
STATS 101
Introductory Statistics

Lecture 5

Continuous Probability Distributions



Discrete and Continuous Random Variables

Examples:
(1) # of Heads in 3 tosses of a fair coin.
(2) Sum of two faces when a pair of fair dice are rolled.

- A discrete random variable (rv) takes integer values only.

A continuous random variable can take any real value inside an interval.

Examples:

(1)Heights (or weights) of kids in a group.

(2) Daily temperatures in Las Vegas

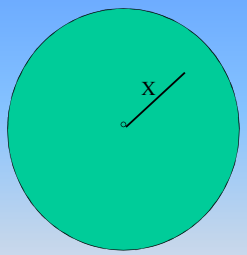
ONE EXAMPLE OF A CONTINUOUS DISTRIBUTION

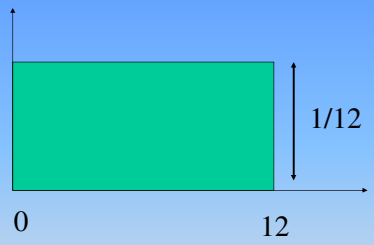
UNIFORM DISTRIBUTION

Example:

Suppose you are throwing darts at a circular dart board of radius 12 inches, completely at random. Assume that you manage to hit the dart board each time. Let

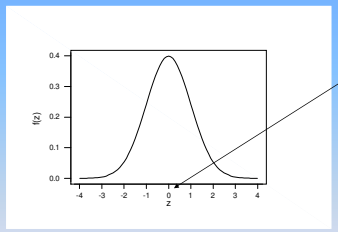
X = distance of dart from the center of the board





Total Probability = Area of green rectangle = 12 x (1/12) = 1

THE MOST COMMONLY USED CONTINUOUS DISTRIBUTION

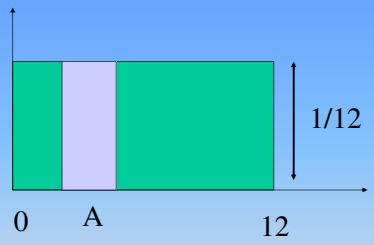


Mean = 0

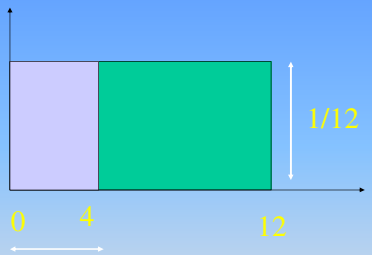
sd = 1

= spread

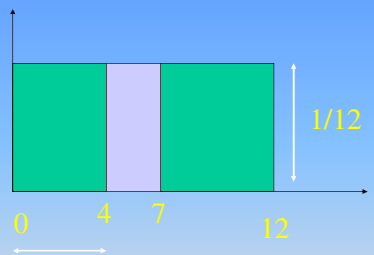
STANDARD NORMAL DISTRIBUTION



Probability of interval A= Area of gray rectangle /area of green rectangle

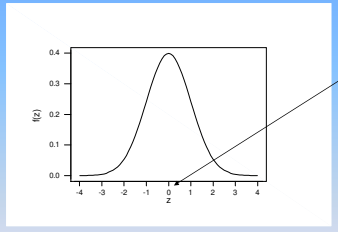


P(X < 4 inches) = 4/12 = 1/3



P(4<X < 7 inches) = 3/12 = 1/4

THE MOST COMMONLY USED CONTINUOUS DISTRIBUTION

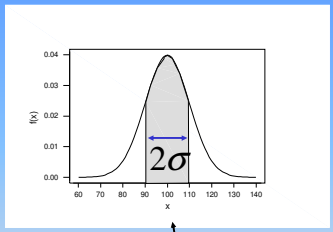


Mean = 0

sd = 1

= spread

STANDARD NORMAL DISTRIBUTION



Mean = 100

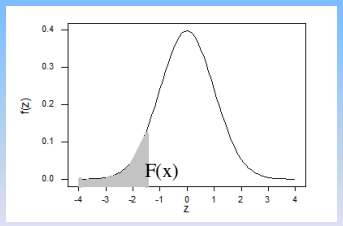
Normal distribution N(100, sd = 10)

Probability Density Function of a Continuous Random Variable

$f(x) \geq 0$ such that

$P(X \leq x)$ = shaded area under the density function

F(x) = shaded area = Cumulative Distribution Function (cdf)



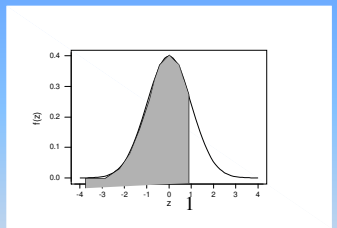
x

Calculating Probabilities for

The Standard Normal Distribution

Example 1: Z ~ N(0,sd=1)

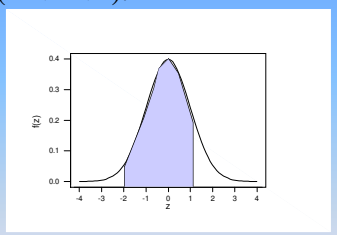
Find P(Z < 1).



P(Z < 1) = F(1) = gray area = 0.8413 from the standard normal probabilities table

Example 2: Z ~ N(0,sd=1)

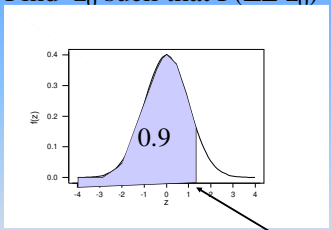
Find P(-2<Z < 1).



P(-2<Z < 1) = F(1) – F(-2) = 0.8413 – 0.0228 = 0.8185

Example 3: Z ~ N(0,sd=1)

Find z_0 such that $P(Z \leq z_0) = 0.90$



From the standard normal table, the z-value with cdf 0.8997 equals 1.28.

Calculating Probabilities for

The General Normal Distribution

Example 1: X ~ N(100,sd=10)

Find P(90<X < 110).

RESULT: $P(X \leq x) = P(Z \leq \frac{x-\mu}{\sigma})$

where Z = standard normal N(0,1)

P(90<X < 110) is calculated as follows:

$$P(90 < X < 110)$$
$$= P(X \leq 110) - P(X \leq 90)$$
$$= P(Z \leq \frac{110-100}{10}) - P(Z \leq \frac{90-100}{10})$$
$$= P(Z \leq +1) - P(Z \leq -1)$$
$$= 0.8413 - 0.1587 = 0.6826$$

Probability corresponding to +1

Probability corresponding to -1

Example 2: X ~ N(100,sd=10)

Find the value x_0 such that

$P(X \leq x_0) = 0.9$

$$P(X \leq x_0) = P(Z \leq \frac{x_0 - 100}{10}) = 0.90$$

From the table of normal probabilities, the probability of 0.90 gives

$$z_0 = \frac{x_0 - 100}{10} = 1.28$$

Solve the above equation for x_0:

$x_0 = 100 + 10 \times 1.28 = 112.8$